



Section 2

new cost function is $\frac{1}{2} \sum (y_i - \hat{y}_i)^2$

- Choosing The orientation, shown in x_1 - x_2 plane $(0,0), x_2$

- Line 1 $\rightarrow (0,1), (1,0)$

$$\frac{x_1 - 0}{x_2 - 1} = \frac{1 - 0}{0 - 1}$$

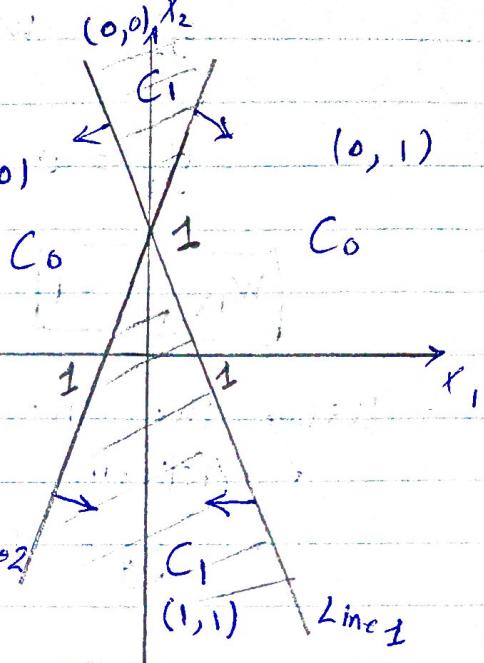
$$-x_1 = x_2 - 1$$

$$x_1 + x_2 - 1 = 0$$

testing the orientation using $(0,0)$ Line 2

↳ negative \rightarrow Changing the orientation

$$-x_1 - x_2 + 1 = 0 \quad \text{Line 1}$$



- Line 2 $\rightarrow (0,1), (-1,0)$

$$\frac{x_1 - 0}{x_2 - 1} = \frac{0 - (-1)}{1 - 0}$$

$$x_2 - 1 = x_1$$

$$x_1 - x_2 + 1 = 0$$

testing the orientation using $(0,0)$

↳ positive

$$x_1 - x_2 + 1 = 0 \quad \text{Line 2}$$

- We have two separation lines

- So we need ~~2~~ to two neurons in the hidden layer

- Using the neuron N_3 to represent the Line 1

* Activation of N_3

$$y_3 = w_{13}x_1 + w_{23}x_2 + w_{03}$$

$$y_3 = 0 \rightarrow [-x_1 - x_2 + 1 = 0]$$

$$w_{13} = -1$$

$$w_{23} = -1$$

$$w_{03} = 1$$

- Using the neuron N_4 to represent the Line 2

* Activation of N_4

$$y_4 = w_{14}x_1 + w_{24}x_2 + w_{04}$$

$$y_4 = 0 \rightarrow [x_1 - x_2 + 1 = 0]$$

$$w_{14} = 1$$

$$w_{24} = -1$$

$$w_{04} = 1$$

- to provide the required classification with the chosen orientation, we need to perform XNOR operation on the output of neurons N_3, N_4

- We Design AND Gate using neuron N_5

- Activation of N_5

$$y_5 = w_{35}f(y_3) + w_{45}f(y_4) + w_{05}$$

	$f(y_3)$	$f(y_4)$	$f(y_3) \cdot f(y_4)$	$f(y_3) \cdot f(y_4) + w_{05}$
	0	0	0	1
	0	1	0	0
	1	0	0	0

- for $f(y_3) = 0, f(y_4) = 0$

$$y_5 = [w_{05} \neq < 0]$$

- for $f(y_3) = 0, f(y_4) = 1$

$$y_5 = [w_{45} + w_{05} < 0]$$

- for $f(y_3) = 1, f(y_4) = 0$

$$y_5 = [w_{35} + w_{05} < 0]$$

- for $f(y_3) = 1, f(y_4) = 1$

$$y_5 = [w_{35} + w_{45} + w_{05} > 0]$$

We Choosing $\rightarrow y_5 = f(y_3) + f(y_4) - 1.5$

$$w_{35} = 1$$

$$w_{45} = 1$$

$$w_{05} = -1.5$$

The Activation of neuron N6

$$y_6 = w_{36} f(y_3) + w_{46} f(y_4) + w_{56} f(y_5) + w_{06}$$

\rightarrow for $f(y_3) = 0, f(y_4) = 0 \rightarrow f(y_5) = 0 \rightarrow s = 1$

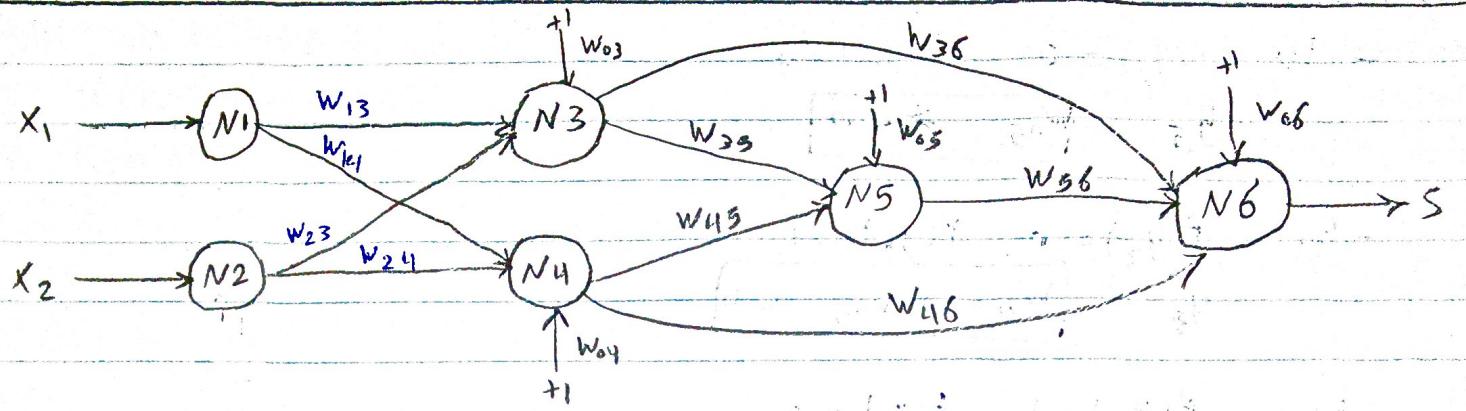
$$y_6 = [w_{06} > 0]$$

\rightarrow for $f(y_3) = 0, f(y_4) = 1 \rightarrow f(y_5) = 0 \rightarrow s = 0$

$$y_6 = [w_{46} + w_{06} < 0]$$

\rightarrow for $f(y_3) = 1, f(y_4) = 0 \rightarrow f(y_5) = 0 \rightarrow s = 0$

$$y_6 = [w_{36} + w_{06} < 0]$$



Using Binary threshold Function for hidden and output neurons

→ for $f(y_3) = 1, f(y_4) = 1 \rightarrow f(y_5) = 1 \rightarrow S = 1$

$$y_6 = [w_{36} + w_{46} + w_{56} + w_{66} > 0]$$

We choose $\rightarrow y_6 = -1.5 f(y_3) - 1.5 f(y_4) + 2.5 f(y_5) + 1$

$$w_{06} = 1$$

$$w_{36} = -1.5$$

$$w_{46} = -1.5$$

$$w_{56} = 2.5$$

x_1	x_2	y_3	$f(y_3)$	y_4	$f(y_4)$	y_5	$f(y_5)$	y_6	$S=f(y_6)$	Classification
0	2	-1	0	-1	0	-1.5	0	1	1	C_1
0	-2	3	1	3	1	0.5	1	0.5	1	C_1
2	1	-2	0	2	1	-0.5	0	-0.5	0	C_0
-3	0	4	1	-2	0	-0.5	0	-0.5	0	C_0

- No, we can't classify $(1, 2) \rightarrow$ because it lies on the Line 2

$$x_1 - x_2 + 1 = 1 - 2 + 1 = 0$$